Price Clustering and Discreteness: Is there Chaos behind the Noise?

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Abstract

We investigate the "compass rose" (Crack, T.F. and Ledoit, O. (1996), Journal of Finance, 51(2), pg. 751-762) patterns revealed in phase portraits (delay plots) of stock returns. The structures observed in these diagrams have been attributed mainly to price clustering and discreteness. Using wavelet based denoising, we examine the noise-free versions of a set of FTSE100 stock returns time series. We reveal evidence of non-periodic cyclical dynamics. As a second stage we apply Surrogate Data Analysis on the original and denoised stock returns. Our results suggest that there is a strong nonlinear and possibly deterministic signature in the data generating processes of the stock returns sequences.

1 Introduction

The empirical investigation of the dynamics of stock returns has been an area of intensive research since the beginning of last century (see thesis of Bachelier (1)). The understanding of dynamics observed in price fluctuations are of paramount importance to activities such as forecasting for investment decision support, risk modelling and derivative pricing. Moreover, the complexity of their structure, as a result of agent-market interactions, is an indicator of the nature of overall

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market conditions and organization. This complexity may also reflect the level of agent's rationality and risk tolerance. It becomes apparent that the explanation of certain qualities of the structure of market dynamics, provides the opportunity to improve the understanding of their current and future states. Clearly such an exercise is of great importance to all market participants that aim to minimize their risks and protect their investments and profits.

Viewing economies and markets in particular as a dynamical system, we can draw many inferences by examining their observable outputs: sequences of stock prices and the corresponding returns. Crack and Ledoit (2) have first revealed a "*compass rose*" pattern discovered in scatter diagrams of returns against their lagged values (i.e., phase portraits), such as the one depicted in Fig. 1(a). They attributed the pattern to price clustering and discreteness and especially the tick size and suggested reasons for its appearance.[1] Our aim is by using an approach consistent with the tradition of econophysics, to continue their research by revealing yet more interesting patterns and showing that the compass rose is a mask for more subtle dynamics. In this paper we establish the case of existence of nonstochastic nonlinear dynamics via the calculation of the BDS statistic (8). We use this as a discriminating statistic for a permutation test based framework ("Surrogate Data Analysis" (SDA) by (9)) that allows us to support our results at various levels of significance. As a second step, following (10; 11; 12), we reduce the level of noise in the original returns sequences using Wavelet based thresholding (the Waveshrink technique by (13)). We then recalculate the BDS statistic on the denoised sequences and their surrogates and test again for the absence of linear dynamics. Meanwhile we produce the compass rose of the denoised sequences only to reveal an entirely different structure that is strongly reminiscent of a dynamical attractor. Our findings are consistent with the hypothesis that the returns sequence dynamics may be characterized by nonlinearities that can be of a complex-deterministic character. The results produced here may bring us closer to establishing that a significant part of the driving force generating financial prices could indeed be chaotic.

[Insert figure 1 about here.]

[1] Clustering in stock market prices has been an issue that concerned research since the 1960's (e.g., see refs. (3; 4) who were motivated by the original findings of (5)). Ref. (6) investigated dependencies related to clustering and discreteness. This research was followed by (7) who conducted simulations on price rounding and discreteness and showed that the hypothesis of a geometric Brownian motion for daily and weekly frequencies could be rejected. In general, price clustering and discreteness is an important chapter of "market's microstructure" with serious implications on risk evaluation, the optimal design of securities and market efficiency.

2 Investigating the Compass Rose

Crack and Ledoit (2) suggested first the use of phase portraits in order to reveal the compass rose. This implied the investigation of some sort of time-dependency among stock return sequences. This could be linear or nonlinear, a result of stochastic (random) or nonstochastic (deterministic) data generating process (DGP), or even a mixture of the above behind the asset price dynamics. The authors also proposed that the formations revealed could be of use for calibrating tests of the existence of chaos in returns sequences. Since then various papers have appeared on this theme (see (14; 15; 16; 17; 18; 19; 20; 21)). We believe that two issues can be addressed further:

1. As (22) note, observed stock prices are not always the true equilibrium prices and hence the image of market dynamics observed through them could be partial. Moreover, in markets where significant fixing takes place, there is a variable amount of error introduced into the price level which is then passed to the returns ((see 23, for a discussion on this)).

2. Generating logarithmic or percentage returns, i.e., 1st order differencing, is a *high-pass* filter (24). In this respect, all return sequences will contain amplified noise. Consequently, any interesting and possibly non-stochastic structures may be concealed and/or distorted. The importance of this becomes even greater if we take into account point (1) above.

In the following pages, we investigate further the issue of compass rose formations in stocks from the UK market. We analyze the daily closing prices of stocks in the FTSE ALL SHARE and especially the FTSE 100 index, spanning the period 01/01/1970 to 5/30/2003 (a maximum of 8717 observations). A total of 53 FTSE100 stocks were available with a full (homogeneous) range of prices for the above time-span. Remarkably, all 53 high-capitalization company prices and corresponding returns revealed the patterns we observe and report in this paper (some more intensively and clearly than others).[2]

3 Surrogate Data Analysis and Waveshrink

Following (9; 25) (see also (26; 27)), we investigated the possibility of the observed structures of the compass rose being a "one-off" situation. The basic purpose of the SDA procedure is to provide a framework that will allow us to deny the null hypothesis that the data are generated by a linear stochastic system. It basically comprises of two steps (see (28; 27) for an extensive overview):

[2]Although (2) use percentage returns, we concentrate on continuously compounding return sequences (logarithmic returns) and observe the same patterns.

- The production of data sets from a model which captures deliberately only certain "linear" properties of the original sequence. These sets are called "surrogate data".

- The rejection of the null hypothesis H_0 according to a calculation of a discriminating statistic. This will suggest that the original data is very unlikely to have been generated by a process consistent with the null hypothesis.

If the value of the statistic calculated on the original data set is different from the sets of values obtained on the surrogate data, we have a clear indication for the rejection of the null. There are various different nulls, some more composite than others and each null is usually accompanied by its own procedure of surrogate data generation. For the purposes of this paper we followed Refs. (27; 29; 25). We thus generated phase-randomized amplitude-adjusted surrogates (termed "AAFT") to test for the null hypothesis that the return sequences were monotonic nonlinear transformation of linearly filtered noise (which is also maintained as the "most interesting"). Such surrogates are expected to exhibit the same spectral and distributional characteristics as in the original series, however they are purely linear processes. As a discriminating statistic we chose the BDS test (8; 30; 31; 32).

We simulated AAFT surrogate data from the original returns sequences, and produced the compass roses for various stocks. An example of an AAFT surrogate set compass rose for the BP stock is presented in Fig. 3(c). We can clearly see there that both the randomly shuffled sequence (Fig. 3(b)) and the AAFT surrogates loose the compass rose structure whereas the bootstrapped sequence maintains it (Fig. 3(d)).[3] This was an initial indication that the results of clustering and discreteness may not be manifestations of linear-random dynamics.

Following the results of the SDA analysis on the phase portraits, we chose to test for independence under an SDA framework for a subset of 53 FTSE100 stocks' returns. We used the BDS test as a discriminating statistic, and generated the AAFT surrogate sets for each stock, testing the null at 5%, 2.5% and 1% significance levels. In tables 1 and 2, we present the results of the SDA. In table 1 we quote the results for a BDS test neighborhood size of 0.5 times the standard deviation of each returns sequence, for significance levels $\alpha = 5\%$, 2.5% and 1%. The results here refute clearly the null that the sequences are a monotonic nonlinear transformation of linearly filtered white noise. This is a strong indication of absence of linear dynamics and randomness and supports the premise of nonlinear deterministic complexity in the returns. The results of table 2 are also supporting this finding. There we have provided more detail, checking for neighborhood sizes of $\epsilon_1 = 0.5 \times s_x$, $\epsilon_2 = 1.0 \times s_x$, $\epsilon_3 = 1.5 \times s_x$ and $\epsilon_4 = 2.0 \times s_x$, where s_x denotes the standard deviation of each returns sequence. The level of

[3]For a discussion on the differences of bootstrapping and surrogate data analysis refer to (33).

significance for table 2 is $\alpha = 5\%$. The results clearly show that the above null is strongly refuted.

[Insert Table 1 about here.]

[Insert Table 2 about here.]

[Insert Table 3 about here.]

Since the results of SDA where pointing towards more complex, nonlinear dynamics (possibly deterministic) we tested as a next step, the returns sequences after these have been filtered for noise reduction. For each stock returns sequence we produced a filtered version, using the Waveshrink (13) approach. We then produced AAFT surrogates and tested for $\alpha = 2.5\%$ significance level. In table 3 we produce the results for the BP stock, where the Waveshrink (34; 13) routine has been applied for a Daubechies 8 (D8) wavelet.[4]. Wavelets here are a justified choice in order to avoid the "bleaching" of the returns sequences (35), and preserve any delicate deterministic structures in the DGPs. Our approach is also consistent with Refs. (36; 37; 38). Looking at the values of the BDS statistic for the original prefiltered sequence and its AAFT surrogates, as well as the p-value of the statistic for sizes of neighborhood ranging from 0.5 to 1.5 times the standard deviation, we can safely reject the null at a 5% significance level. Only for a size of neighborhood of 2× standard deviation $\epsilon_4 = 0.0035$ (which is a considerable size), we can reject the null at a level of significance of almost 70%.

Searching for qualitative evidence of deterministic dynamics and aperiodic cycles we looked at the phase portraits of the denoised sequences. For example, in Fig. 1 (b) we can clearly see the phase portrait for the BP denoised returns reveals dynamics that are similar to chaotic attractors. A detail of the core of the phase portrait in Fig. 1 (c) exhibits dynamics that are very similar to that of the Mackey-Glass attractor (39) in Fig. 1 (d). This appears to be in line with (40; 41).

[Insert figure 2 about here.]

[4] Choices of different mother wavelets produced similar results. See also Ref. (11)

[Insert figure 3 about here.]

Another interesting diagram that reveals the effects of stock price clustering and discreteness is depicted in Fig. 2 (a). There we have plotted the prices of BP stock against the corresponding logarithmic returns. We can clearly see patterns of correlation and anticorrelation in the same diagram. This is the first time such patterns have been revealed in financial literature and they need to be investigated further. In nonlinear science, the phase portraits (i.e., the compass rose) are usually called "delay plots" whereas the plot of a sequence of prices from a function gainst its first derivative are called "phase plots". Thus the diagram in Fig. 2 (a) could be loosely termed as a phase plot. If we generate the same kind of display for the denoised sequences (in this case for the BP stock), we see clearly the cyclical but aperiodic behavior observed in the phase portraits also repeated here (Fig. 2 (b)).

The results lead us to deduce that the presence of chaotic dynamics can not be excluded. Such a statement though should also involve the calculation of certain invariant measures that characterize chaos (such as entropy or dimension based statistics). Moreover, these results should also be backed by a suitable SDA testing exercise. We retain this as a strategy for future research. It would also be interesting to observe if these smoother though irregular cyclical dynamics revealed in this paper are irrespective of the noise reduction technique (i.e., robust under different noise reduction techniques).

4 Conclusions and future research

We have investigated the dynamics of sequences of daily closing prices and the corresponding returns for stocks traded in the London Stock Exchange in the last three decades, as these are observed through the compass rose phase portraits. Our results suggest that the amount of noise inherent in the examined sequences may be covering more "interesting" dynamics. Using wavelet based noise reduction techniques we filtered the return sequences only to uncover a strong aperiodic nonlinear behavior, characteristic of many phenomena that are governed by complex deterministic dynamics. The SDA hypothesis testing framework employed here also suggests the absence of stochastic randomness and linear dynamics for both original and denoised returns sequences. Our results show that the apparently random dynamics and discreteness observed in closing price sequences, may conceal via the generation of noise in the returns, a more delicate structure and aperiodic cyclical dynamics. However, further research in needed to maintain the hypothesis of nonlinear determinism in stock price time series dynamics.

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Table 1: Surrogate Data Analysis results on actual returns for 53 companies in the FTSE100. Discriminating statistic: BDS test. Neighbourhood size $\epsilon = 0.5 \times s_x$, where s_x = standard deviation of x. Biases and standard errors (s.e.) reported for significance levels $\alpha = 5\%$, 2.5% and 1%.

		$\alpha = 5\%$		$\alpha = 2.5\%$		$\alpha = 1\%$	
	BDS Statistic	bias	s.e.	bias	s.e.	bias	s.e.
FTSE ALL SHARE - PRICE INDEX	27.11	−25.29	1.01	−25.38	1.14	−25.13	1.05
FTSE 100 - PRICE INDEX	31.9	−31.61	1.24	−31.41	1.05	−31.82	0.91
ALLIED DOMECQ	18.68	−18.64	1.08	−18.83	1.1	−18.65	0.94
AMVESCAP	32.51	−32.38	0.7	−31.86	0.98	−32.25	0.90
ASSD.BRIT.FOODS	28.44	−28.44	0.96	−28.26	1.01	−28.35	0.90
AVIVA	22.57	−22.18	0.98	−22.43	1.06	−22.45	1.03
BARCLAYS	23.1	−22.29	1.06	−22.40	1.02	−22.45	0.98
BOC GROUP	23.19	−23.19	1.11	−23.16	1.01	−22.97	1.03
BOOTS GROUP	19.99	−19.55	1.27	−19.45	1.06	−19.46	1.04
BP	17.27	−16.95	1.13	−16.97	0.9	−17.03	1.08
BRIT.AMERICAN TOBACCO	17.76	−17.43	1.23	−17.57	1.12	−17.53	0.97
BRITISH LAND	36.57	−36.36	1.27	−36.19	0.97	−36.57	1.09
BUNZL	25.95	−24.82	0.78	−24.68	1.32	−24.95	1.17
CADBURY SCHWEPPES	24.89	−24.55	1.26	−24.44	0.93	−24.36	1.00
DAILY MAIL 'A'	34.58	−34.10	1.19	−34.21	1.22	−34.12	1.04
DIAGEO	23.41	−23.33	1.11	−23.23	1.11	−23.31	0.99
DIXONS GP.	25.12	−24.65	0.98	−24.54	1.25	−24.72	1.09
EMAP	33.47	−32.82	0.9	−33.04	1.13	−33.04	1.01
EXEL	31.92	−30.59	1.27	−30.88	1	−30.97	1.05
FOREIGN & COLONIAL	25.76	−25.04	0.8	−25.23	1.1	−25.35	1.02
GKN	22.41	−22.37	0.89	−22.43	1.06	−22.22	1.03
GLAXOSMITHKLINE	18.48	−18.05	0.94	−18.04	1.14	−18.27	1.13
GRANADA	31.42	−30.44	1.16	−30.35	0.95	−30.42	1.11
GUS	43.46	−43.23	1.29	−43.28	0.9	−43.31	1.16
HANSON	23.26	−22.95	1.19	−22.58	1	−22.76	0.99
HILTON GROUP	22.41	−22.23	1.1	−22.15	1.05	−22.12	0.92
IMP.CHM.INDS.	21.56	−21.43	0.96	−21.50	1.04	−21.35	1.05
JOHNSON MATTHEY	28.6	−27.32	0.9	−27.76	1	−27.61	1.13
LAND SECURITIES	26.82	−26.36	0.79	−26.31	1.15	−26.33	1.05
LEGAL & GENERAL	24.83	−24.67	1.23	−24.67	1.08	−24.62	1.03
MARKS & SPENCER GROUP	22.22	−22.10	1.22	−22.25	0.9	−22.26	1.06
MORRISON (WM) SPMKTS.	22.36	−21.71	1.22	−21.91	1.11	−21.86	1.08
NEXT	23.44	−23.05	1.05	−23.05	0.9	−22.85	1.06
PEARSON	29.34	−28.47	1.05	−28.56	0.78	−28.63	1.15
PROVIDENT FINL.	32.23	−31.93	1.13	−31.53	1.03	−31.45	1.04
PRUDENTIAL	23.3	−23.13	0.87	−23.12	0.95	−23.14	0.96
RECKITT BENCKISER	23.43	−22.62	1.29	−22.62	0.76	−22.45	1.00
REED ELSEVIER	24.67	−24.40	1.37	−24.29	0.93	−24.43	0.92
RENTOKIL INITIAL	29.41	−28.93	0.78	−29.09	1.07	−28.91	1.04
REXAM	26.26	−26.08	1.03	−25.63	1.1	−25.97	1.05
RIO TINTO	23.14	−22.48	0.8	−22.58	1.21	−22.52	1.15
ROYAL BANK OF SCOTLAND	27.18	−26.85	1.11	−26.83	0.92	−26.81	1.11
SAINSBURY (J)	27.13	−26.78	0.92	−26.66	1.07	−26.62	0.99
SCHRODERS	32.63	−32.39	1.17	−32.45	1.05	−32.46	0.95
SCOT. & NEWCASTLE	28.46	−27.93	1.25	−28.43	1.08	−28.27	1.05
SHELL TRANSPORT & TRDG.	24.07	−23.36	0.96	−23.69	0.86	−23.47	1.05
SMITH & NEPHEW	28.28	−28.07	0.74	−27.76	0.98	−27.78	0.91
SMITHS GROUP	25.67	−24.30	0.99	−24.00	1.14	−24.07	0.89
STD.CHARTERED	33.79	−32.78	0.71	−33.03	0.92	−32.88	1.09
TESCO	20.95	−20.58	1.12	−20.53	0.78	−20.49	0.92
TOMKINS	27.42	−27.36	0.95	−27.57	0.89	−27.42	1.05
UNILEVER (UK)	23.95	−23.70	0.81	−23.25	1.14	−23.48	1.03
WHITBREAD	22.32	−21.59	1.14	−21.64	0.82	−21.81	1.06
WOLSELEY	26.37	−25.03	1.2	−25.16	1.13	−25.50	1.00
WPP GROUP	34.11	−33.73	0.86	−33.80	0.94	−33.64	0.91

Table 2: Surrogate Data Analysis results on actual returns for 53 companies in the FTSE100. Discriminating statistic: BDS test (embedding dimension 3). Neighbourhood size $\epsilon_1 = 0.5 \times s_x$, $\epsilon_2 = 1.0 \times s_x$, $\epsilon_3 = 1.5 \times s_x$ and $\epsilon_4 = 2.0 \times s_x$, where s_x = standard deviation of x. Biases and standard errors reported for significance level $\alpha = 1\%$.

	Statistic (BDS)				Bias				Standard Error			
Neighbourhood size	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_1	ϵ_2	ϵ_3	ϵ_4
FTSE ALL SHARE - PRICE INDEX	20.8	23.87	26.81	28.72	−18.61	−21.45	−24.32	−26.37	1.11	1.13	1.17	1.23
FTSE 100 - PRICE INDEX	21.25	15.31	15.27	13.02	−21.41	−15.44	−15.41	−13.09	1.1	0.92	0.87	0.99
ALLIED DOMECQ	13.73	15.09	16.38	17.17	−13.72	−15.01	−16.25	−16.99	1.02	0.97	0.99	1
AMVESCAP	25.6	24.1	22.33	21.13	−25.13	−23.55	−21.74	−20.52	0.95	0.89	0.91	1
ASSD.BRIT.FOODS	22.73	24.5	23.7	22.53	−22.50	−24.33	−23.49	−22.41	1.06	1	1.04	1.08
AVIVA	18.31	19.37	19.79	20.26	−18.44	−19.37	−19.75	−20.18	1.11	1.18	1.14	1.11
BARCLAYS	18.25	20.46	22.67	23.97	−17.82	−19.74	−21.82	−23.12	1.04	1.09	1.03	0.98
BOC GROUP	18.3	17.06	16.67	16.13	−18.25	−16.93	−16.48	−16.00	0.91	1.02	1.06	1.01
BOOTS GROUP	15.77	16	17.12	18.31	−15.19	−15.32	−16.44	−17.74	1	1.01	0.99	0.99
BP	12.89	13.62	13.8	14.07	−12.68	−13.36	−13.44	−13.69	1.02	0.94	0.93	0.95
BRIT.AMERICAN TOBACCO	14.28	16.3	16.96	17.67	−14.50	−16.34	−16.91	−17.55	1.07	1.21	1.19	1.13
BRITISH LAND	28.42	29.97	30.8	32.2	−28.28	−29.80	−30.44	−31.82	1.01	1.01	1.01	1.05
BUNZL	21.76	19.84	18.87	15.15	−20.55	−18.16	−17.00	−13.30	1.03	0.9	0.99	1.19
CADBURY SCHWEPPES	20.53	22.39	23.32	23.81	−20.00	−21.82	−22.68	−23.11	0.97	1.05	1.04	1.1
DAILY MAIL ‘A’	28.24	28.42	26.45	22.3	−27.93	−27.91	−25.90	−21.72	1.14	1.26	1.22	1.07
DIAGEO	18.05	18.36	18.96	19.04	−17.58	−17.75	−18.32	−18.45	1.18	1.15	1.17	1.12
DIXONS GP.	20.6	20.62	20.17	19.48	−19.95	−19.95	−19.59	−19.08	1.14	1.09	0.95	0.84
EMAP	27.63	22.23	19.17	17.41	−27.25	−21.93	−18.94	−17.17	1.04	1.03	1.08	1.08
EXEL	25.56	23.08	19.81	17.98	−24.13	−21.34	−18.02	−16.34	0.78	1.03	1.18	1.2
FOREIGN & COLONIAL	20.47	19.83	21.56	22.04	−19.97	−19.22	−20.87	−21.46	0.96	0.97	0.98	0.99
GKN	17.02	18.68	18.42	17.65	−16.55	−18.15	−17.85	−17.07	1.04	1.04	1.13	1.16
GLAXOSMITHKLINE	14.12	15.45	16.11	16.3	−13.89	−15.17	−15.77	−15.95	0.89	0.89	0.82	0.93
GRANADA	24.34	26.02	24.88	22.38	−23.31	−24.73	−23.46	−21.01	1.05	1.06	1.06	0.99
GUS	29.9	25.99	24.7	22.03	−29.71	−25.71	−24.45	−21.82	1.02	1.03	0.92	0.87
HANSON	19.56	19.93	19.53	18.07	−18.57	−18.89	−18.55	−17.22	1.07	1.13	1.1	1.11
HILTON GROUP	17.77	18.17	18.48	19.22	−17.17	−17.28	−17.44	−18.27	1.02	1.01	1.04	1.03
IMP.CHM.INDS.	17.11	18.92	19.68	19.58	−17.28	−18.84	−19.45	−19.38	1.05	1.01	1.03	1.05
JOHNSON MATTHEY	22.63	21.34	20.24	17.27	−21.60	−19.93	−18.77	−15.94	1.09	1.13	1.08	0.99
LAND SECURITIES	21.54	22.95	24.18	25.44	−20.93	−22.33	−23.57	−24.93	0.9	0.9	0.87	0.92
LEGAL & GENERAL	19.17	20.94	22.65	23.96	−18.98	−20.72	−22.36	−23.57	1.13	1.08	1.04	1.07
MARKS & SPENCER GROUP	17.69	19.03	20.36	20.98	−17.93	−19.25	−20.58	−21.24	0.95	0.92	0.85	0.8
MORRISON (WM) SPMKTS.	19.14	18.52	17.87	15.61	−18.71	−17.76	−17.14	−15.00	1.16	0.93	0.94	0.97
NEXT	17.35	18.96	21.43	21.21	−16.96	−18.41	−20.83	−20.68	1	1.06	1.11	1.08
PEARSON	23.95	24.97	23.55	22.58	−22.81	−23.65	−22.22	−21.38	0.92	0.91	0.94	1.04
PROVIDENT FINL.	23.8	22.06	20.06	18.6	−23.10	−21.24	−19.10	−17.76	1.19	1.08	0.96	0.94
PRUDENTIAL	18.21	19.09	21.01	22.67	−17.52	−18.38	−20.40	−22.19	0.86	0.92	1.02	1.1
RECKITT BENCKISER	19.09	20.73	21.89	22.29	−18.03	−19.43	−20.45	−20.91	1.06	0.93	0.92	0.97
REED ELSEVIER	19.57	19.57	19.44	18.94	−19.15	−19.02	−18.87	−18.47	0.79	0.9	1.04	1.03
RENTOKIL INITIAL	23.14	21.06	21.56	20.11	−22.77	−20.42	−20.90	−19.50	0.84	0.99	1.06	1.02
REXAM	20.18	19.56	18.95	18.24	−19.89	−19.18	−18.50	−17.66	0.87	0.95	1.02	1.02
RIO TINTO	17.96	18.65	18.84	18.71	−17.02	−17.46	−17.57	−17.42	0.85	0.84	0.96	1.06
ROYAL BANK OF SCOTLAND	21.44	21.5	22.06	21.63	−21.15	−21.16	−21.64	−21.08	1.06	1.06	1.02	0.99
SAINSBURY (J)	21.44	22.58	23.46	22.49	−21.16	−22.18	−22.85	−21.79	0.9	0.91	0.96	1.04
SCHRODERS	25.54	27.25	26.83	25.09	−25.43	−27.05	−26.39	−24.92	1.17	1	1.08	0.91
SCOT. & NEWCASTLE	21.71	21.8	22.32	22.31	−21.38	−21.37	−21.82	−21.76	0.86	0.86	0.98	1.06
SHELL TRANSPORT & TRDG.	19.4	20.45	20.78	20.38	−19.14	−20.11	−20.39	−20.00	1	1	1.17	1.24
SMITH & NEPHEW	22.05	22.23	20.99	20.91	−21.76	−21.77	−20.43	−20.34	1.12	1.11	1.03	1.06
SMITHS GROUP	20.72	20.23	19.2	18.16	−18.91	−17.98	−16.78	−15.94	1.02	1.11	1.09	1.03
STD.CHARTERED	26.61	25.67	24.1	21.95	−25.44	−24.23	−22.63	−20.53	0.97	1.14	1.17	1.21
TESCO	16.39	15.57	15.6	16.05	−16.14	−15.20	−15.06	−15.44	1	1.18	1.19	1.22
TOMKINS	21.92	14.76	15.41	13.84	−21.79	−14.55	−15.25	−13.67	1.07	0.92	1.02	0.95
UNILEVER (UK)	19.48	20.62	20.74	19.9	−18.71	−19.76	−19.96	−19.30	0.83	0.88	0.94	0.92
WHITBREAD	16.91	17.65	17.6	17.21	−16.26	−16.79	−16.73	−16.37	0.89	0.84	0.9	0.94
WOLSELEY	20.51	19.06	17.85	16.49	−19.71	−17.82	−16.45	−15.20	1.14	1.01	0.88	0.88
WPP GROUP	27.81	22.97	20.73	23.13	−27.38	−22.23	−19.98	−22.17	1.06	1.02	1.3	1.34

Table 3: Surrogate Data Analysis results on D8 pre-filtered BP returns. Discriminating statistic: BDS test (embedding dimension 2). Neighbourhood size $\epsilon_1 = 0.5 \times s_x$, $\epsilon_2 = 1.0 \times s_x$, $\epsilon_3 = 1.5 \times s_x$ and $\epsilon_4 = 2.0 \times s_x$, where s_x = standard deviation of x.

	Neighbourhood Size			
set	$\epsilon_1 = 9e-04$	$\epsilon_1 = 0.0017$	$\epsilon_2 = 0.0026$	$\epsilon_4 = 0.0035$
1	922.89	444.06	316.55	280.90
2	837.02	417.54	300.76	266.65
3	934.51	446.17	317.84	281.37
4	933.59	446.77	318.31	281.97
5	880.90	430.70	308.58	273.29
6	936.90	446.80	318.40	281.44
7	889.16	432.69	309.08	272.54
8	928.31	444.19	316.63	279.94
9	916.11	441.64	316.03	280.41
10	881.64	431.72	308.36	273.35
11	916.71	441.76	315.27	279.04
12	932.64	446.76	318.92	282.99
13	832.50	417.22	300.04	266.62
14	932.27	446.08	318.41	282.52
15	925.76	444.06	317.08	280.87
16	941.02	447.37	318.04	279.56
17	913.54	440.37	314.19	278.49
18	888.79	433.23	309.91	274.35
19	935.24	446.81	318.27	281.28
20	831.79	416.32	299.63	265.58
21	799.68	406.98	292.95	259.33
22	832.68	416.32	298.47	264.23
23	865.91	425.52	305.02	269.74
24	884.16	432.43	310.04	275.97
25	882.36	431.60	309.22	274.49
26	872.69	427.79	306.57	271.06
27	903.62	437.21	312.30	276.44
28	943.29	449.77	320.33	284.10
29	927.47	444.60	317.18	280.81
30	897.28	435.32	310.77	274.88
31	931.34	446.29	318.95	283.01
32	892.41	434.36	311.11	276.10
33	921.26	441.78	315.33	277.76
34	882.30	432.09	309.36	274.94
35	938.17	448.33	319.22	282.41
36	935.55	447.50	318.72	282.32
37	809.99	409.72	295.84	262.73
38	919.59	442.14	315.87	279.80
39	885.79	432.02	309.67	274.05
40	877.49	430.08	308.29	273.56
Original	1793.68	567.17	337.70	273.50
Significance	23.07	11.43	3.61	0.40
p-value	0.00	0.00	0.00	0.69

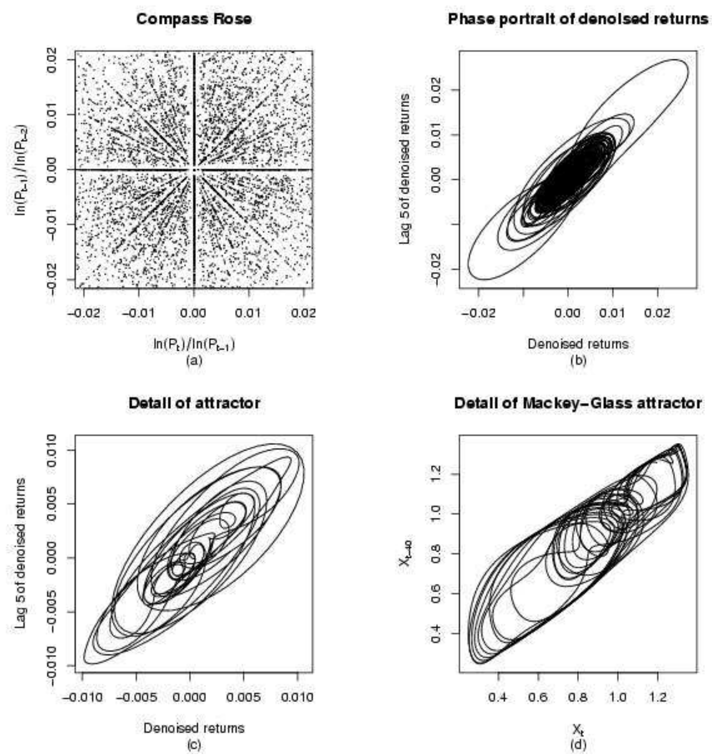

Figure 1: The BP stock returns compass rose (a), details of the denoised BP returns phase portraits (b,c) and the Mackey-Glass attractor (d).

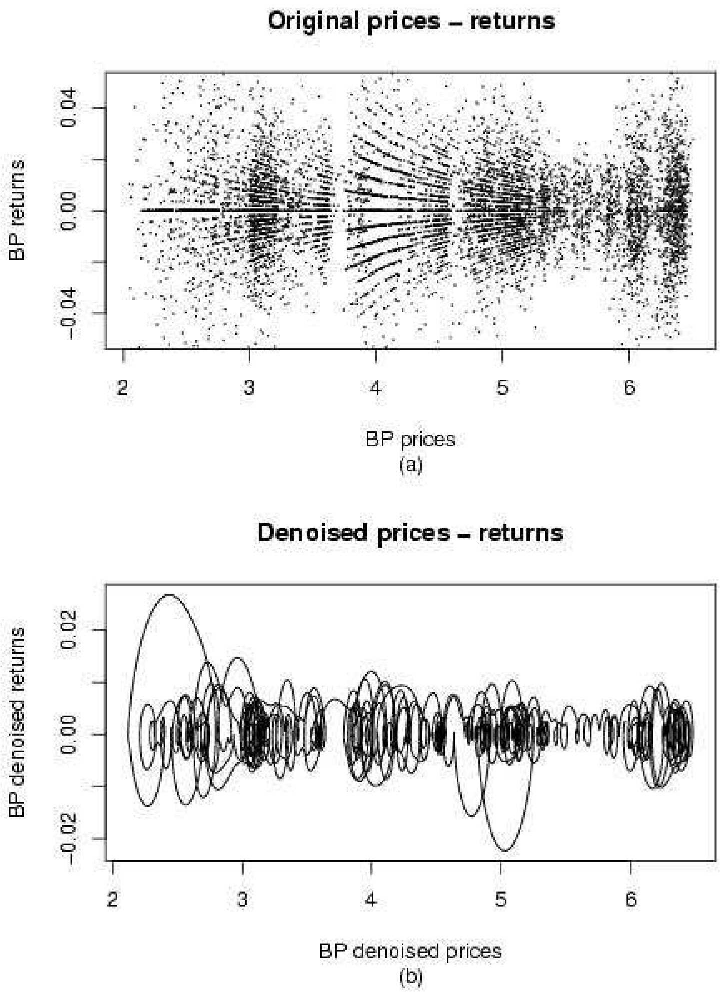

Figure 2: Phase diagrams of the original (a) and denoised (b) BP prices-returns.

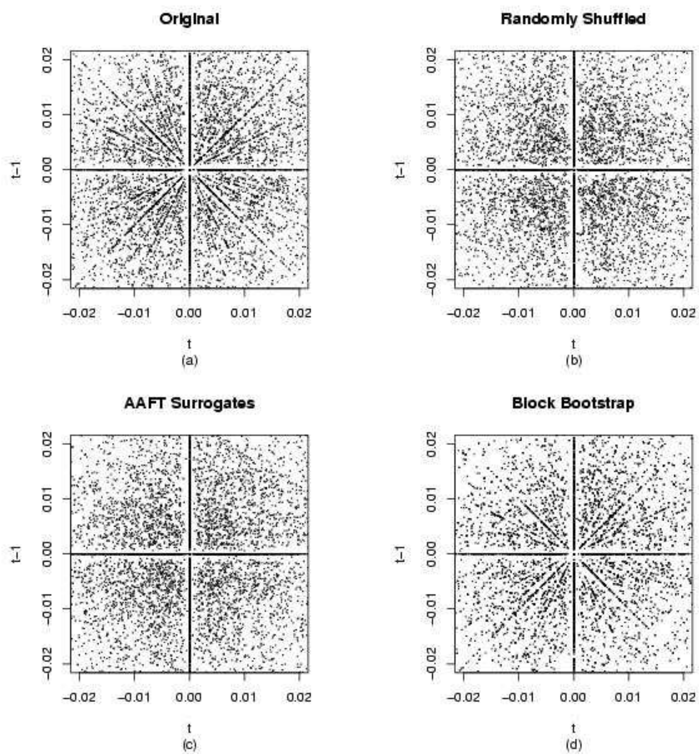

Figure 3: Details of compass roses of the original (a), randomly shuffled (b), AAFT surrogate (c) and bootstrapped (d) BP returns sequences.